Modal Epistemic Logic
with Subjunctive Markers
and Knowability

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A diffuse philosophical tendency cannot be refuted once and for all by a single rigorous argument. Nevertheless, such an argument can severely constrain the forms in which the tendency is expressed.

(Timothy Williamson)

The ‘diffuse philosophical tendency’: Anti-Realism

The ‘rigorous argument’: The Church-Fitch Paradox of Knowability

The anti-realistic Knowability Principle (KP):

“Every truth is knowable!”
The Church-Fitch Paradox

(1) $\alpha \rightarrow \Diamond K\alpha$  \[\text{[Formalisation of KP]}\]

(2) $\Box (K\alpha \rightarrow \alpha)$  \[\text{[Factivity of knowledge]}\]

(3) $\Box (K(\alpha \land \beta) \rightarrow (K\alpha \land K\beta))$  \[\text{[Knowledge distributes over conjunction]}\]

(4) $\neg \Diamond K(\alpha \land \neg K\alpha)$  \[\text{[from (2) and (3)]}\]

(5) $(\alpha \land \neg K\alpha) \rightarrow \Diamond K(\alpha \land \neg K\alpha)$  \[\text{[from (1)]}\]

(6) $\neg (\alpha \land \neg K\alpha)$  \[\text{[from (4) and (5)]}\]

(7) $\neg K\alpha \rightarrow \neg \alpha$  \[\text{[from (6)]}\]

(8) $\alpha \rightarrow K\alpha$  \[\text{[from (7)]}\]
Varieties of Anti-Realism

a) Very Soft Anti-Realism

(Withdrawing from KP or restricting it)

b) Soft Anti-Realism

(Staying with KP but denying (7) and (8))

c) Moderately Hard Anti-Realism

(Accepting (7), but not (8))

d) Very Hard Anti-Realism

(Accepting that every truth is known)
The standard conception of knowability:

Idea:
Maybe there is another sensible conception of knowability and consequently another sensible reading of KP...
Modal logic with subjunctive markers:

**Scope theory of the subjunctive mood:**
The subjunctive mood indicates that the expression has to stand inside modal scope. The indicative mood indicates that the expression has to stand outside modal scope.

*Problem:*
The scope theory is false!

*Counterexample:*
“Under certain counterfactual circumstances everyone who actually has flown to the moon would not have flown to the moon.”
Solutions to the expressibility problem:

- best solution:
  Wehmeier’s modal logic with subjunctive markers

\[ \diamondsuit_{s1} \forall_x (F^i x \rightarrow \neg F^{s1} x) \]

- standard solution:
  Modal logic with actuality operator

\[ \diamond \forall_x (A F x \rightarrow \neg F x) \]

The knowability principle revisited:

\[ \alpha \rightarrow \diamondsuit_{s1} K^{s1} \alpha \quad \text{[within modal logic with subjunctive markers]} \]

\[ \alpha \rightarrow \diamond K A \alpha \quad \text{[within modal logic with actuality operator]} \]
Blocking the Church-Fitch derivation

(1*) \( \alpha \rightarrow \diamondsuit_{s1} \ K^{s1}_1 \alpha \)

(2*) \( \Box_{s1} (K^{s1}_1 \alpha \rightarrow \alpha) \)

(3*) \( \Box_{s1} (K^{s1}_1 (\alpha \land \beta) \rightarrow (K^{s1}_1 \alpha \land K^{s1}_1 \beta)) \)

(4*) \( \neg \diamondsuit_{s1} K^{s1}_1 (\alpha \land \neg K^{s1}_1 \alpha) \)

(5*) \( (\alpha \land \neg K\alpha) \rightarrow \diamondsuit_{s1} K^{s1}_1 (\alpha \land \neg K\alpha) \)

Edgington’s conception of knowability:

Problem:
This conception just doesn’t work!
(Mainly because of Williamson’s trivialisation arguments)
Epistemic logic with subjunctive markers:

Motivation:

The Bush-Kerry example

Two aspects of knowledge:

(1) How knowledge is (or might be) expressed by sentences or propositions
   \[\Rightarrow\] knowledge \textit{de dicto}

(2) The subject matter (objects, relations) knowledge is about
   \[\Rightarrow\] knowledge \textit{de re}
**Formalisation:**

Not only modal operators bind subjunctive markers, but epistemic operators bind subjunctive markers, too!

knowledge *de dicto*
\[ \Rightarrow \] The expressions in the scope of the knowledge operator are in the subjunctive mood

knowledge *de re*
\[ \Rightarrow \] The expressions in the scope of the knowledge operator are in the indicative mood
Example:

Fa  “Nowitzki is blond”

$K_{s1} F^{i} a^{i}$  It is known about Nowitzki (in the de re sense) that he is blond (in the de re sense). Maybe somebody knows the following proposition: “The German NBA player has light hair”

$K_{s1} F^{s1} a^{i}$  It is known about Nowitzki (in the de re sense) that he is blond (in the de dicto sense). Maybe somebody knows the following proposition: “The German NBA player is blond”

$K_{s1} F^{i} a^{s1}$  It is known that Nowitzki (in the de dicto sense) is blond (in the de re sense). Maybe somebody knows the following proposition: “Nowitzki has light hair”

$K_{s1} F^{s1} a^{s1}$  Somebody knows the proposition: “Nowitzki is blond”
Combining modal logic with subjunctive markers and epistemic logic with subjunctive markers:

Each subjunctive marker can either be bound by a modal operator or by an epistemic operator.

Result:

Modal epistemic logic with subjunctive markers
The knowability principle revisited (part II):

Possible readings of the knowability principle:

(1) $\alpha^i \rightarrow \diamond s_1 K_{s_2}^{s_1} \alpha^i$ [Rückert’s proposal for the anti-realist]

(2) $\alpha^i \rightarrow \diamond s_1 K_{s_2}^{s_1} \alpha^{s_2}$ [the standard conception; leads to the Church-Fitch Paradox]

(3) $\alpha^i \rightarrow \diamond s_1 K_{s_2}^{s_1} \alpha^{s_1}$ [a third reading; leads to similar problems as (2)]
Rückert’s conception of knowability:

Concluding remarks

THAT’S IT!!!

⇒ Discussion