Abstract. There is an argument (first presented by Fitch), which tries to show by formal means that the anti-realistic thesis that every truth might possibly be known, is equivalent to the unacceptable thesis that every truth actually is known (at some time in the past, present or future). First, the argument is presented and some proposals for the solution of Fitch’s Paradox are briefly discussed. Then, by using Wehmeier’s modal logic with subjunctive marker ($S5^*$), it is shown how the derivation can be blocked if one respects adequately the distinction between the indicative and the subjunctive mood. Essentially, this proposal amounts to the one by Edgington which was formulated with the help of the actuality-operator. Finally it is shown how the criticisms by Williamson against Edgington can be answered by the formulation of a new conception of possible knowledge that $\alpha$ (thereby $\alpha$ being in the indicative mood and thus referring to the actual world). This conception is based on the concept of same $de\ re$ knowledge in different possible worlds.

Anti-realists such as Dummett and Wright claim that linguistic meaning is intimately related to the use of relevant expressions by the human linguistic community. What is expressed by a certain sentence thus depends essentially on how it is used. According to this position it is not possible that the states of affairs that are expressed might be in principle independent from the corresponding contexts of use that may arise in the linguistic community:

The meaning of a mathematical statement determines and is exhaustively determined by its use. The meaning of such a statement cannot be, or contain as an ingredient, anything which is not manifest in the use made of it, lying solely in the mind of the individual who apprehends that meaning: if two individuals agree completely about the use to be made of the statement, then they agree about its meaning. (Dummett 1978, 216)

Thus, it is plausible to accept the thesis that there are no states of affairs that may be expressed by linguistic means and that are nevertheless in principle inaccessible to the members of the linguistic community. This means for the anti-realistic concept of truth that it is epistemically constrained: Truths have to be epistemically accessible to the members of the linguistic community in principle. In its strongest reading this condition says that every truth might under certain (possibly counterfactual) circumstances also be known. Now, an ordinary language formulation of the anti-realistic thesis (ART) can be given:

(ART) Every truth might possibly be known.
1. The Derivation of Fitch’s Paradox

If one wants to give (ART) a formal reading, the following formula (which reflects the redundancy of the truth predicate for the purposes at stake and which uses schematic letters instead of explicit quantification over propositions) seems to be the first choice:

\[ \alpha \rightarrow \Diamond K\alpha \]  

(1)

\( K \) is the knowledge operator meaning something like “it is known by somebody at some point in time, that…”, and the concept of modality at issue is a metaphysical one and not a mere epistemic one.

If anti-realism is understood according to thesis (1), it has to face a serious problem: There is a simple argument – it was firstly presented by (Fitch 1963), and has become much more popular through (Hart 1979) – that aims to show, that (1) implies the much more problematic thesis that every truth at some point in time actually is known, that there are no unknown truths.

For the derivation two more premises are needed. First, the unproblematic assumption about the concept of knowledge which says that it is a necessary condition of knowledge that it is factive. If something is known then it has to be true:

\[ \Box (K\alpha \rightarrow \alpha) \]  

(2)

Also it is assumed that knowing a conjunction necessarily implies that both conjuncts are known:

\[ \Box (K(\alpha \land \beta) \rightarrow (K\alpha \land K\beta)) \]  

(3)

From (2) and (3) it follows logically, that it is impossible to know that something is a never known truth. Assume this would be possible: \( \Diamond (K\alpha \land \neg K\alpha) \). Because of (3) it follows that \( \Diamond (K\alpha \land K\neg K\alpha) \). By help of (2) again we get \( \Diamond (K\alpha \land \neg \alpha) \), a contradiction. Thus:

\[ \neg \Diamond (K\alpha \land \neg K\alpha) \]  

(4)

Now, by substituting in (1) \((\alpha \land \neg K\alpha)\) for \( \alpha \) we get to

\[ (\alpha \land \neg K\alpha) \rightarrow \Diamond K(\alpha \land \neg K\alpha). \]  

(5)

From (4) and (5) follows (6):

\[ \neg (\alpha \land \neg K\alpha). \]  

(6)
And (6) again is (at least in classical logic) equivalent to

\[ \alpha \rightarrow K\alpha. \] 

(7)

The derivation of Fitch’s Paradox is completed. Apparently it has been shown that the thesis that every truth might possibly be known implies the thesis that every truth actually is known at some time or other. From (1) to (7) a modal collapse happened: the possibility operator has disappeared. And even if anti-realists want to accept thesis (1) they usually don’t want to accept (7). Because to claim that everything which is true also is known at some time or other seems to be at least very problematic if not absurd.

Even if what (4) says is correct, namely that of no proposition is it possible to know that it is true and also that it is never known, one nevertheless wants to accept the existential claim that there are propositions which are true but actually (because of contingent reasons) never known. For example, it is impossible for us to know of something lying outside our light cone (and that it lies outside of our light cone is a contingent fact), because – if Einstein’s theory of relativity is correct in this respect – we can’t get into causal contact with such facts.

Thus, the anti-realist has a problem. How should he react to Fitch’s Paradox? But, in my opinion, also his enemy, the realist, should be interested in an adequate reaction, because even if he rejects (ART), it seems dubious that the dispute about an apparently interesting difficult philosophical thesis might be decided with the help of simple logical means by showing that (ART) implies the unacceptable (7). Also to the realist an intuitive understanding of (ART) can be assigned, that shows (ART) to be less strong and problematic than (7).5

At the very least the anti-realist should have a good response to Fitch’s Paradox at his disposal in order to defend his philosophical position. Several different starting points for an anti-realist reaction are intelligible. Some of them will be discussed shortly in the following. On the reaction to Fitch it also depends how the respective anti-realist position has to be formulated in more detail. Certainly, one has to agree with Williamson when he writes:

A diffuse philosophical tendency cannot be refuted once and for all by a single rigorous argument. Nevertheless, such an argument can severely constrain the forms in which the tendency is expressed. (Williamson 2000, 99)

2. Possible Reactions to Fitch’s Paradox

Confronted with a philosophical-logical argument of the present kind, four different strategies might be followed:

(i) One accepts the conclusion. But then, one has to show that the conclusion is, against the first impression, nevertheless acceptable or compatible with the defended position. This would mean for the anti-realist, who is surprisingly
willed to accept (7), that he has to show that it is, according to his position, plausible to hold the claim that every truth actually is known at some time or other.

(ii) One doubts that the derivation of the conclusion from the premises is correct. To approach the problem in this way, one has to show that one or more argumentative steps haven’t been well-founded.

(iii) One rejects one or more of the premises. For the following the principles (2) and (3), which seem to be highly plausible assumptions about the concept of knowing, will not be called into question. This leaves us with the anti-realistic thesis. Here, a distinction should be drawn:

(A) One could defend that (ART) represents adequately anti-realism but that its formal representation with (1) is not correct, and (1) should be replaced by another formula of the underlying logic.

(B) The other possibility consists in claiming that the acceptance of already (ART) is not forced by a correctly understood anti-realistic position.

(iv) One doubts the suitability of the used instruments. In our case the used instruments are standard modal logic and the knowledge operator $K$. At first sight, this strategy seems to be hopeless, because standard modal logic is very well established and only very few and unproblematic assumptions concerning the $K$-operator are made use of. But who knows?

In the following, several responses (reflecting all four strategies) to Fitch’s Paradox will be discussed. In this section we will discuss proposals that can be related to strategies (i) to (iii). In the subsequent section the main proposal of this paper is presented, performing a combination of strategies (iii)(A) and (iv).6

2.1. The Ideal Epistemic Subject: A Way Out?

First, let’s discuss the consequent acceptance of thesis (7). Following a terminological proposal by Tennant (1997, 261), we will call an anti-realist, who is ready to accept (7) a ‘very hard anti-realist’. Suppose, as suggested above, that the totality of human knowledge never can comprise the wealth of actually existing truths (and a healthy amount of modesty as well as a realistic appraisal of the limitations of our cognitive capacities seem to lead to this conclusion), which way out is still available not to be forced to withdraw from (7)?

One might think of a similar (theological) move as (Berkeley 1710) had imagined in order to stick with the esse est percipi of his radical subjective idealism without having to draw the absurd consequence that objects cease to exist as soon as they are no longer perceived by human beings. In analogy to the objects’ always being perceived by God in Berkeley the very hard anti-realist could retain thesis (7) by claiming that even if not every truth is known (at some time or other) by a human being, every truth is known by a postulated ideal epistemic subject or an omniscient God. And thus, (7) would be true nevertheless.7
What to think about this theological way out? Leaving aside any problems the hypothesis of an omniscient God or a similar conception might bring with it and whether one’s own attitude towards such a doctrine is positive or negative, it is clear that this proposal completely misses the point of the debate that concerns us. The original anti-realistic argument referred to a (our) human linguistic community and aimed to show, that every truth might possibly be known by a (possible) member of this community. This means that the $K$-operator contains an implicit double existential quantification, one about points of time (in the past, present or future) and the other about the members of the communication community. Let’s make this explicit by using $Mx$ as an abbreviation for “$x$ is a member of the linguistic community”, $Tx$ as an abbreviation for “$x$ is a point of time”, and the doubled indexed knowledge operator $K_{xy}\alpha$ meaning “$x$ knows at $y$, that $\alpha$”:

$$K\alpha \iff \exists x \exists y (Mx \land Ty \land K_{xy}\alpha)$$

By help of this more detailed definition of the $K$-operator it is now obvious that the strategic move, the idea that God knows all truths, is no promising defence of thesis (7), because, so to speak, God is not accepted as a potentially relevant knowing subject (he doesn’t fulfil the predicate M).8

2.2. INTUITIONISTIC LOGIC

The philosophical position of anti-realism arose when intuitionistic and constructivist tendencies in the philosophy of mathematics were generalised and applied to other areas.9 In the field of mathematics the movement coincided with a replacement of classical logic by intuitionistic logic. Thus, it would be quite consequent for the anti-realist to use intuitionistic logic also in his generalised project. Here, we will discuss this proposal very briefly, because the debates in the newer literature about Fitch’s Paradox mainly deal with the possibilities for a solution resulting from the acceptance of intuitionistic logic (sometimes accompanied by a weakening of thesis (1)). And also, the main aim of this paper is to propose a different solution.10

First, using intuitionistic logic seems to offer another way of defending the acceptability of thesis (7) which was derived by Fitch’s argument. Consequently, if one uses intuitionistic logic, one also should use one of the semantics developed for intuitionistic logic. Thus, one can give the conditional in (7) an intuitionistic reading. One such reading says that an intuitionistic conditional is correct if everyone who knows the antecedent is thus in a position to know the consequent, too. Understood in this way, (7) seems to be fully acceptable.11

The principle problem for this strategy (leaving aside the question whether an consistent formulation of the main thesis without unwanted side-effects can be given12) is that if the anti-realist wants to give thesis (7) a special intuitionistic meaning of the mentioned kind (by giving an intuitionistic reading to the
conditional), (7) does no longer serve as a translation of the acceptable ordinary-
language proposition that not all truths are known by human beings at some time or
other, and the anti-realist needs to present another formula of his intuitionistic lo-
gical language to express this proposition. Because a position which doesn’t allow
to do so seems to be at least very dubious.

A further starting point for a possible solution of Fitch’s Paradox amounts from
the question whether all the steps in its derivation are also intuitionistically valid
and not only classically (as known, intuitionistic logic is weaker than classical
logic). Indeed, the derivation till
\[ \neg(\alpha \land \neg K\alpha) \] (6)
is completely correct, also in an intuitionistic setting. But the remaining step leading to
\[ \alpha \to K\alpha \] (7)
is only acceptable from a classical point of view. For the anti-realist using intu-
itionistic logic this fact opens the possibility to stick with (ART) and (1), but to
withdraw from (7), because (7) has been derived by intuitionistically unacceptable
means. But then still he has to accept (6) as well as the thesis (8) that follows
intuitionistically from (6):
\[ \neg K\alpha \to \neg \alpha \] (8)

To come again back to Tennant’s terminology we will call an anti-realist willing to
accept (6) and thus (8) as correct, but not (7), a ‘moderately hard anti-realist’.\textsuperscript{13}

Translated into ordinary language (8) says something like that everything that
is not known at some time or other also is not true. And at first sight, this seems
to be almost as absurd as what is said by (7). Once again, one might try to justify
(8) by giving a special intuitionistic interpretation to the logical particles contained
in (8), for example by giving the conditional one of the readings discussed above.
This strategy leads to the problem that such a reading not only allows to justify
the acceptance of (8), but that of (7) as well, and thus moderately hard anti-realism
turns out to be a very hard anti-realism. This is not the place to engage in a more
detailed discussion of moderately hard anti-realism, but it seems to me that this
position is not very natural with regard to the ideas that in the beginning did lead
to anti-realism (cf. Dummett’s argument from the beginning of this paper). Thus,
an anti-realist should not be happy with moderately hard anti-realism unless there
is no other consistent and useful alternative at his disposition.

The aim of Sections 3 and 4 of this paper is to show that ‘soft anti-realism’,
accepting (ART) but neither (7) or (8), can be successfully defended against Fitch’s
argument.\textsuperscript{14}
2.3. Considering Time-Parameters

According to the analysis of the $K$-operator given above using $K$ involves an implicit existential quantification over the members of the linguistic community as well as an implicit existential quantification over all points of time (in the past, present or future). Propositions containing the $K$-operator thus also refer to the future. Already Aristotle (1975, 18a–19b), at hand of a sentence saying that there would be a sea battle the next day, discussed the position that (contingent) sentences about the future don’t have a truth-value yet, but that they get one as soon as the point of time they talk about arises. Thus, sentences about the future are no real propositions which are either true or false. And indeed, the thesis that sentences about the future already now have a definite truth-value seems to presuppose the correctness of the metaphysical thesis of determinism, which is called into question by current interpretations of modern quantum physics.

Let’s examine whether the adoption of the Aristotelian position leads to a solution of Fitch’s Paradox. Instead of using $K$ we introduce a knowledge operator $K_t$, which says something like “there is a point in time $t’ \leq t$, and at $t’$ it is known that...”, where $t$ may lie in the past or the present but not in the future. Furthermore, $K_t \alpha$ may only be true if $\alpha$ does not refer to a point in time lying in the future with respect to $t$, in which case $\alpha$ would not already have a truth-value at $t$. Thus, we introduce a time index: $\alpha_t$ says that $\alpha$ is true at $t$. Also let the present point of time with respect to our world be named by $t_0$.

After these preparations we are now able to give the anti-realistic thesis the following formulation which takes time-parameters into consideration:

$$\alpha_{t_1} \rightarrow \Diamond K_{t_2} \alpha_{t_1}$$

where there is at least one such $t_2$ with $t_2 \geq t_1$ (9)

But, is this new formulation safe from Fitch? Let’s chose $\alpha = \alpha_{t_0} \land \neg K_{t_0} \alpha_{t_0}$.

Substitution in schema (9) results in:

$$(\alpha_{t_0} \land \neg K_{t_0} \alpha_{t_0})_{t_1} \rightarrow \Diamond K_{t_2} (\alpha_{t_0} \land \neg K_{t_0} \alpha_{t_0})_{t_1}$$

(10)

On the basis of the assumptions made, it is immediately clear that it has to be $t_1 = t_0$, and thus apparently also $t_2 = t_0$. It seems as if (10) results in (11):

$$(\alpha_{t_0} \land \neg K_{t_0} \alpha_{t_0})_{t_0} \rightarrow \Diamond K_{t_2} (\alpha_{t_0} \land \neg K_{t_0} \alpha_{t_0})_{t_0}$$

(11)

It is easy to check whether starting from (11) Fitch’s Paradox can be derived in the usual way (all time-indices coincide and thus no step in the argument will be blocked with the help of them). Do we have to conclude that the consideration of time-parameters does not lead to a way out?

No, we don’t have to draw this conclusion. But first, some more modifications are necessary. In the argument leading us from (10) to (11) at the stage that it apparently has to be that $t_2 = t_0$ we used the condition that $t_2$ must not lie in the
future, but that it has to be that \( t_2 \geq t_1 \). But now, if \( t_1 = t_0 \), then \( t_2 = t_0 \). But the expression \( K_{t_2}(\alpha_{t_0} \land \neg K_{t_0}\alpha_{t_0})_{t_1} \) lies within the scope of a modal operator and thus has to be semantically evaluated always with respect to the corresponding possible worlds at stake. Thus, it results only necessarily that \( t_2 = t_0 \) if the present point of time with respect to each possible world is identical with \( t_0 \).

What still needs to be done in order to block Fitch’s derivation with the help of the consideration of time-parameters is a determination of the concept “possible world” where this does not have to be so. Let for each possible world \( w_i \) the corresponding present point in time be named by \( t_{w_i} \). Then, a world \( w_m \) is a possible alternative with respect to \( w_n \) if and only if the following conditions are fulfilled:

1. \( t_{w_m} \geq t_{w_n} \)
2. \( \{ \alpha \mid \alpha \text{ is true at } w_n \} \subseteq \{ \alpha \mid \alpha \text{ is true at } w_m \} \)
3. \( w_n \) and \( w_m \) have to fulfil the usual conditions for possible worlds (in particular, the sets of truths of the different worlds have to be consistent)

(C1) and (C2) together amount to the fact that a possible alternative to a given possible world always has to comprise the complete course of this world (that is the class of true propositions in this world), and (apart from the limit case that the starting world and the possible alternative are identical) develops it further to a certain future point in time (the present point in time with respect to that possible alternative). (C3) demands the further condition of internal consistency of all possible worlds. For example, it excludes that in a world \( w \) somebody knows \( \alpha \), but \( \alpha \) isn’t even true (this would contradict the factive character of knowledge).

The modal logic resulting from the above considerations obviously is of the S4-type, because the accessibility relation between possible worlds \( R \) has to be reflexive and transitive (but isn’t symmetric as soon as the frame contains more than one possible world). What remains to be shown is that the anti-realistic thesis (9) is now no longer subject to an argument \( à la \) Fitch. Let’s consider again (10), which resulted from (9) by the substitution, that is characteristic for Fitch’s Paradox:

\[
(\alpha_{t_0} \land \neg K_{t_0}\alpha_{t_0})_{t_1} \rightarrow \Diamond K_{t_2}(\alpha_{t_0} \land \neg K_{t_0}\alpha_{t_0})_{t_1}
\]  

(10)

Because \( t_0 \) is the present point of time with respect to the real world and because sentences about the future don’t have a truth-value yet, it follows that it has to be that \( t_1 = t_0 \):

\[
(\alpha_{t_0} \land \neg K_{t_0}\alpha_{t_0})_{t_0} \rightarrow \Diamond K_{t_2}(\alpha_{t_0} \land \neg K_{t_0}\alpha_{t_0})_{t_0}
\]  

(12)

Would \( t_2 = t_0 \) Fitch’s Paradox could again be derived, because for no member of the linguistic community is it possible to know at \( t_0 \) that the problematic conjunction is true at the same moment (the second conjunct says that no member of the linguistic community knows the first conjunct at \( t_0 \)). But the problem is resolved in case of \( t_2 > t_0 \), because then a member of the linguistic community might know at a future point in time \( t_2 \), that the conjunction was true at \( t_0 \), which includes
knowledge of the fact that nobody (himself included) knew at \( t_0 \) that \( \alpha \) was true at \( t_0 \) (see Figure 1).

Now, it should be clear that the assumptions made about the relation between truth-values and time-parameters as well as the given concept of possible world, allow to deal successfully with Fitch’s Paradox. But the assumptions one has to concede in order to go this way are not unproblematic. First of all one has to adopt the Aristotelian view that (contingent) sentences about the future don’t have a truth-value yet and thus are no real propositions. This leads to some linguistic hardship cases because one is forced, for example, to declare the reply “Yes, that’s true” to the assertion “Tomorrow Man.U. plays in the Champion’s League”, which might arise very naturally in ordinary discourse, to be not fully correct.

Thus, even if the assumptions made might be defended, the proposed analysis does not represent a real alternative for anti-realists like Wright, because they would have to give up the so-called ‘timelessness of truth’-thesis and would have to accept several restrictions of the usual modal logic framework, whereas modern anti-realists tend to be closer to realistic positions with respect to these matters.

2.4. MELIA’S LIBERALISATION OF (ART)

In a very short note (Melia 1991) has argued that the anti-realistic thesis (1) as well as its ordinary language formulation (ART) shouldn’t be accepted by the anti-realist, because the claims thus expressed were too strong, and therefore could be exploited to derive Fitch’s Paradox. According to Melia’s analysis there is an improper assumption (about the independence of the truth-value of a proposition
from the attempts to find out this truth-value) which creeps in the initial argument that led to (ART):

This argument presupposes that we can always discover a statement’s truth value without affecting that statement’s truth value. But this is not so: there exist statements which are true, yet which would have been false had we performed the procedures necessary to discover that statement’s truth value. (Melia 1991, 341)

Of course, Melia thinks in particular of propositions of the type $\alpha \land \neg K\alpha$ as used in the derivation of Fitch’s Paradox. These propositions are constructed in such a way that if they are true they cannot be known, because in one conjunct it is expressed that the other conjunct (and thus the whole conjunction) is not known.

Thus, if we tried (in an alternative possible world) to find out the truth-value of the proposition in question, we certainly would start by trying to find out the truth-value of $\alpha$. Two cases have to be distinguished here: Would $\alpha$ turn out to be false, we would know that the conjunction is false, too. Would $\alpha$ turn out to be true, the whole conjunction would turn out to be false as well (because then the second conjunct would be false) and we would have reached a case, called ‘pathological’ by Melia, in which only the counterfactual circumstances resulting from the fact that it is tried to find out the truth-value of the conjunction, would lead to a change of this truth-value from ‘true’ (in the real world) to ‘false’ (in the alternative possible world).

Considerations of this kind lead Melia to the following conclusions concerning the position of anti-realism:

For the anti-realist can surely distinguish between (i) statements whose truth value changes whenever we try to verify them, and (ii) statements whose truth value cannot be discovered no matter how we try to verify them. The anti-realist takes exception to statements belonging to class (ii), but I see no reason why that should make him reject statements in class (i). (Melia 1991, 342)

Melia considers the demands resulting from (ART) to be unjustified, and claims that the anti-realist should withdraw from (ART) and replace it with a more liberal thesis. Even if Melia does not formulate explicitly such a thesis of his own, his remarks justify ascribing the following liberalised anti-realist thesis (LART) to him:

\begin{quote}
\textbf{(LART)} \textit{For every proposition there are (possible) circumstances under which it is known which truth-value this proposition has under these circumstances}
\end{quote}

An anti-realism resulting from a substitution of (ART) by a weaker thesis like (LART) we call a ‘very soft anti-realism’, thus expanding Tennant’s terminology. (LART) can be adequately rendered by formal means as:

\[ \Diamond (K\alpha \lor \neg K\neg \alpha) \quad (13) \]

Now, the question arises whether it is possible (similar to Fitch’s Paradox) to derive unpleasant consequences from (13) by clever substitutions for $\alpha$ together with
logical means alone. There are propositions that make the first disjunct false, for example those of the form $\alpha \land \neg K\alpha$ (as used in the derivation of Fitch’s Paradox), as well as propositions that make the second disjunct false, for example those of the form $\neg \alpha \lor K\alpha$. In both cases only $\Diamond (\alpha \rightarrow K\alpha)$ can be derived immediately, but this formula seems to be perfectly acceptable. Also it is not possible by truth-functional compositions of such propositions to get to propositions which make the first and the second disjunct false, thus showing the schema (13) to be incorrect.

Let’s assume that Melia’s proposal is immune from logical constructions. How is it to be judged then? The liberalised anti-realistic thesis (LART) is much weaker than (ART), because it is no longer claimed that every aspect of the world as it really is, might in principle be epistemically accessible. Now, the very soft anti-realist would even concede to the realist that the actual world might be constituted in a way that some aspects of it might be in principle unknowable.

Were there no other acceptable proposals to deal with Fitch’s Paradox, Melia’s proposal would maybe be an interesting alternative for the anti-realist, who then would have to weaken his claim concerning the relation between truth and knowability. In the following it is shown that (ART) can be retained, and thus Melia’s proposal loses its attraction for anti-realists. Because, if there are no logical arguments against (ART), the discussion as regards content might start from the stronger anti-realistic thesis.

3. Solving Fitch’s Paradox with the Help of the Distinction between the Indicative and the Subjunctive Mood

The result of the last section was that there are several possible ways of dealing with Fitch’s Paradox. By accepting intuitionistic logic one can try to defend a hard or a moderately hard anti-realism. An Aristotelian neutralism concerning the future also allows for a solution of the difficulty. And finally, as proposed by Melia, one might weaken the anti-realistic thesis (ART), and for example replace it by (LART).

But up to now we still lack (apart from the Aristotelian proposal) a conception which allows to defend what I believe to be the most compelling form of anti-realism, namely soft anti-realism, against Fitch’s Paradox. A soft anti-realist wants to retain (ART), but not accept either (7) or (8). Aim of the following two sections is to show how soft anti-realism can be successfully defended against Fitch. It is clear that then (ART) can no longer be expressed by the formula (1), because (1) logically implies (7) and thus excludes soft anti-realism. Thus, we will have to give the natural language thesis (ART) a modified formal translation. In order to prepare this move, we will first present Wehmeier’s modal logic with subjunctive marker ($S5^*$).
3.1. Wehmeier’s Modal Logic with Subjunctive Marker

Recently, Wehmeier has criticised Kripke’s modal argument (cf. Kripke (1980)) against the so-called Frege and Russell-view of proper names along with standard modal logic, which is implicitly used by Kripke (see (Wehmeier (2003a) and Wehmeier (2003b)). According to his diagnosis the argument does not consider carefully enough the semantically relevant distinction between the modes. While predicates standing in the indicative mood always have to be evaluated with respect to the real world, predicates standing in the subjunctive mood have to be evaluated with respect to the possible worlds at stake. That the difference between the indicative and the subjunctive mood is semantically relevant can easily be shown with two examples:

(a) Under certain counterfactual circumstances the man who would have taught Alexander would not have taught Alexander.

(b) Under certain counterfactual circumstances the man who taught Alexander would not have taught Alexander.

Both expressions only differ in the fact that the subjunctive “would have taught” in (a) is replaced by the indicative “taught” in (b). This difference is semantically relevant: (a) is false, but (b) expresses a truth. Because (a) says that there are possible circumstances under which the one who taught Alexander (under these circumstances) did not teach Alexander (under these circumstances), a logical contradiction. On the other hand, (b) says that there are possible circumstances under which the one who actually taught Alexander, namely Aristotle, would not have taught Alexander (under these circumstances). And that’s perfectly imaginable, because it might have been that Alexander would have been taught by another person than Aristotle, for example by his own father.

But, one may ask, why does this example already speak against standard modal logic? Because standard modal logic has means to deal with the difference between the indicative and the subjunctive mood: If, in a natural language example, a predicate is in the subjunctive mood, this only shows that the proposition has to be rendered formally in such a way that the corresponding predicate of the logical formula stands inside the scope of a modal operator, whereas predicates in the indicative mood have to be rendered by the logical analysis in such a way that they stand outside any modal scope. This conception suggests that the modes of natural language are only means which help to determine the scopes of modal contexts. And as different scopes can easily be dealt with by the help of parentheses it seems not to be necessary to make the difference between the indicative and the subjunctive mood any more explicit in our logical notation.

What to think about this solution to the problem posed by the examples above? Is it always possible to reduce differences in the modes to differences concerning the placement of the parentheses? True, it is easy to explain the difference between (a) and (b) by this manoeuvre. But, there are also propositions that do not allow for such a move:
(c) Under certain counterfactual circumstances everyone who actually has flown to the moon would not have flown to the moon. 

\[ \forall x (F x \rightarrow \Diamond \neg F x) \] 

as the formal rendering of (c) is not correct because according to (c) there must be counterfactual circumstances under which no one of those, who have actually flown to the moon, would have flown to the moon. But this is not required by the formula because it says that for everyone who actually has flown to the moon there have to be circumstances under which he would not have flown to the moon, and these counterfactual circumstances need not to be the same for everyone. Also, no other formula of standard modal logic does the job.25

Thus, it has been shown that usual modal logic has a serious problem, at least if it is conceived of as a tool to deal with the modal part of ordinary discourse. It is not able to provide a formula for all natural language propositions one might expect one. Its expressibility is not high enough to even deal with all simple cases of everyday modal discourse.

To repair these defects, Wehmeier proposes to change the formal language and make the difference between the indicative and the subjunctive mood explicit.26 For our purposes it is not necessary to present the modal logic evolving from this idea in detail. It should be sufficient to discuss the main ideas, which should also give an impression of the exact techniques.

Syntactically, \( S^5 \) behaves like \( S5 \) with the only difference that we add subjunctive versions of all predicates and quantifiers, which are symbolised by an attached subjunctive marker “\(^*\)”.27 Semantically, predicates and quantifiers in the indicative mood are always evaluated with respect to a specially characterised world \( w^* \) (the real world), even if they stand inside the scope of a modal operator. Predicates and quantifiers in the subjunctive mood have to be evaluated, as usual, with respect to those possible worlds as determined by the modal operators in whose scopes they stand.

Essentially the subjunctive marker works as a variable over possible worlds. Thus, every occurrence of “\(^*\)” has to be bound by a modal operator. For the sake of illustration let’s give the translations of all our three examples into \( S^5^* \):

(a\(^*\)) \( \Diamond \neg T^*(a^*x)(T^*x) \)
(b\(^*\)) \( \Diamond \neg T^*(x)(T x) \)
(c\(^*\)) \( \Diamond \forall x (F x \rightarrow \neg F^*x) \)

3.2. The Reformulation of Fitch’s Argument Within \( S^5^* \)

After briefly presenting Wehmeier’s modal logic with subjunctive marker, we will now examine whether Fitch’s argument turns out to be valid also in the \( S^5^* \) framework or whether the derivation will be blocked by the explicit distinction between the indicative and the subjunctive mood.

In \( S^5^* \) thesis (1) changes to:

\[ \alpha \rightarrow \Diamond K^*\alpha \] 

(1\(^*\))
(ART) only requires that what is true, namely $\alpha$, might possibly be known, and not that it is known. That’s why the $K$-operator has to be in subjunctive mood and thus refer to the respective possible world in question and not (necessarily) to the real world. It should already be mentioned that the second occurrence of $\alpha$, although standing inside the scope of a modal operator, does not contain any free subjunctive markers to be bound by this modal operator, because the first occurrence of $\alpha$ does not stand in any modal context and thus all subjunctive markers that might occur in $\alpha$ already have to be bound.

The two necessary conditions about knowledge now look like:

$$\Box(K^*\alpha \rightarrow \alpha)$$  \hspace{1cm} (2*)

$$\Box(K^*(\alpha \land \beta) \rightarrow (K^*\alpha \land K^*\beta))$$  \hspace{1cm} (3*)

Here, it makes no difference whether $\alpha$ and $\beta$ contain any free subjunctive markers that might be bound by the necessity operator, or not. From (2*) and (3*) it follows:

$$\neg\Diamond K^*(\alpha \land \neg K^*\alpha)$$  \hspace{1cm} (4*)

If we now, as in the usual derivation of Fitch’s Paradox, substitute $(\alpha \land \neg W\alpha)$ for $\alpha$ in (1*), we get:

$$(\alpha \land \neg K\alpha) \rightarrow \Diamond K^*(\alpha \land \neg K\alpha)$$  \hspace{1cm} (5*)

But here (4*) is no longer identical with the negation of the consequent of (5*), because the respective second occurrence of the $K$-operator once is in subjunctive mood, and once in indicative mood. That’s why the derivation is blocked and (6) is no longer implied.

Thus, we have now with (1*) a formal rendering of (ART) at hand, that does not imply (6) or even (7), and a consistent formulation of soft anti-realism seems to be in sight.

3.3. MODAL LOGIC WITH SUBJUNCTIVE MARKER VS. MODAL LOGIC WITH ACTUALITY-OPERATOR

Wehmeier’s modal logic with subjunctive marker is intimately related to usual modal logic with a supplementary so-called actuality-operator $A$. 28 The use of indicative versions of predicates, quantifiers and operators produces, that they always have to be evaluated with respect to the real world, no matter whether they stand in the scope of a modal operator, or not. The $A$-operator has a similar effect: If there is an occurrence of the form $A\alpha$, then $\alpha$ gets evaluated with respect to the real world, even if the whole expression $A\alpha$ stands in the scope of a modal operator. Thus, outside any modal context $A\alpha$ and $\alpha$ are equivalent, but inside modal scope the $A$-operator causes, so to say, that $\alpha$ gets extracted from the modal context. The resulting connections are listed in the following table:
Modal logic with A

\[ \alpha \] outside modal scope \ \approx \ \alpha \text{ in indicative mood}

\[ \alpha \] inside modal scope \ \approx \ \alpha^* \text{ in subjunctive mood}

\[ A\alpha \] outside modal scope \ \approx \ \alpha \text{ in indicative mood}

\[ A\alpha \] inside modal scope \ \approx \ \alpha \text{ in indicative mood}

Because of these connections the conjecture that the proposal for a solution of Fitch’s Paradox just formulated within the framework of Wehmeier’s logic could be reformulated by using standard modal logic plus actuality-operator too, suggests itself.

And indeed (Edgington 1985) has made exactly this proposal. The anti-realistic starting thesis gets the following translation by Edgington:

\[ A\alpha \rightarrow \Diamond KA\alpha \] (1’)

Here, the derivation of Fitch’s Paradox fails for analogous reasons as in the case of (1’), and that needs no further demonstration.

As already Edgington knew, the Achilles’ heel of her (and my) proposal is the sequence of signs \( \Diamond KA\alpha \). Because, at first sight, it is not at all obvious, what it should mean to be a merely possible knowledge of \( A\alpha \) (which refers to the actual world). Edgington’s solution (as well as the analogous one in S5*) thus depends on whether a sensible conception of someone in a possible world knowing something that actually is the case can be formulated. This is the point where most criticisms, formulated at different places, start from. But the counter-arguments against Edgington’s attempts to justify the problematic construction by the help of analogies, especially the analogy between “actually” and the indexical “now” put forward in (Williamson 1987) seem to me to be the clearest and most serious ones. Thus, we will examine Williamson’s criticisms in some more detail next.

### 3.4. The Counter-Arguments by Williamson

The first of Williamson’s objections against the proposal to give (ART) the reading (1’) goes as follows:

In one respect, (4) [i.e., (1’); H.R.] is a surprisingly weak form of verificationism. For, as Edgington notes, ‘\( Ap \)’ always entails ‘\( \Box Ap \)’, where ‘\( \Box \)’ abbreviates ‘it is necessary that’. Thus, the only knowledge that (4) requires is of necessary truths. One might expect a robust form of verificationism to insist that at least some contingent truths are knowable. (Williamson 1987, 257)

But, outside modal scope \( A\alpha \) is equivalent to \( \alpha \). So, why not translate the anti-realistic starting thesis by (1’)?

\[ \alpha \rightarrow \Diamond KA\alpha \] (1’)

The natural objection against (1’*) is probably that according to (ART) the same thing that is true should be knowable, and that thus the antecedent of the
anti-realistic thesis should be identical with the expression in the scope of the knowledge operator.

But, let us again have a look at the anti-realistic thesis corresponding to (1′), or (1″) respectively, formulated within the framework of $S5^*$:

$$\alpha \rightarrow \diamond K^\ast \alpha$$

(1″)

Here, Williamson’s objection becomes pointless, because the schema is no longer only about necessary propositions and the antecedent is identical to the expression in the scope of the $K$-operator. Obviously it depends on the chosen formal language in which cases what stands inside the scope of a modal operator is ‘the same’ as outside, and in which cases not. To defend his own modal logic $S5^*$ in this respect against standard modal logic with actuality-operator, we give the word to Wehmeier:

An obvious constraint on any language of modal predicate logic is that its non-modal part should simply be the language of ordinary predicate logic. Therefore, indicative predicates are to be expressed by the ordinary predicate symbols of non-modal predicate logic (which, after all, formalises ordinary, indicative discourse), and it is the subjunctive for which we need to introduce a new notation. This constraint rules out the standard solution to the expressiveness problem, viz. the introduction of an “actually” operator “$A$”. For consider the following example:

(40) Someone has flown to the moon, but under certain counterfactual circumstances, everyone who has flown to the moon would not have flown to the moon.

With an actuality operator, it would have to be formalised as

(41) $\exists x Fx \& \forall x (A(Fx) \rightarrow \neg Fx)$,

but this is clearly not a faithful representation of the ordinary language sentence (40): In (40), there are two occurrences of the indicative predicate “has flown to the moon”, and on both of these occurrences, the predicate has exactly the same semantic function, viz., to refer to how things stand in the real world. There is one occurrence of “would have flown to the moon”, which is syntactically distinguished by being subjunctive, and semantically distinguished by referring not to the real, but to some counterfactual situation. In (41), however, the first occurrence of “has flown to the moon” in (40) is modelled by “$Fx$”, and so is the occurrence of “would have flown to the moon”, whereas the second occurrence of “has flown to the moon” corresponds to “$A(Fx)$”. This does not appear to be a transparent logical analysis – why should the two occurrences of the predicate “has flown to the moon”, which function in precisely the same way semantically, be modelled in two typographically distinct ways, as “$Fx$” and “$A(Fx)$”? And why is the subjunctive predicate “would have flown to the moon”, referring to some counterfactual world, represented in exactly the same way as the indicative predicate “has flown to the moon” (as it occurs first in (40)), which refers to the actual world? (Wehmeier 2003a, 11–12)

As one can easily see the defects of the actuality-operator version mentioned in this very clear citation disappear if the example is translated into Wehmeier’s $S5^*$: Occurrences of identical predicates in the natural language formulation are...
represented identically in the formal language, whereas occurrences of predicates which differ with respect to the indicative/subjunctive-difference are represented differently.

Obviously, against (1°) Williamson’s criticism does not get through. But, apart from that, is his criticism sound if it is thought of to be directed only against (1’)? I think, it is at least doubtful whether the criterion for necessary statements used by Williamson, namely that \( \alpha \) is a necessary truth iff \( \Box \alpha \) is a truth, should be accepted. Because in cases like \( \Box A \alpha \) the necessity operator is freewheeling, so to say, because of the actuality-operator, as the quantifier in \( \forall x \alpha \) is without effect, supposed \( x \) does not appear freely in \( \alpha \). For a discussion of other criteria, in the context of two-dimensional semantics, see Davies and Humberstone (1980).

But the claim that schema (1’) would only refer to necessary propositions is not the main criticism by Williamson against Edgington’s analysis. His main point concerns the problematic conception of non-actual knowledge of something referring to the real world, already mentioned above. Specially, Williamson remarks that Edgington’s analogies between modal and temporal discourse don’t go through, because there might be causal relations between events that take place at different times, but between events that take place in different possible worlds causal relations are principally excluded (cf. Williamson (1987, 257–258)).

But how then can an expression of the form \( \Diamond K A \alpha \) be understood? For it to be true there has to be an alternative possible world, in which somebody knows at some point in time that \( A \alpha \). In the easiest case this possible world is the real world itself. Then, there are no bigger problems. But in most cases, as in Fitch’s Paradox, it is necessary to consider a possible alternative that is different from the real world, and in which \( A \alpha \) is known. Then, the problem arises how the non-actual knowing subject may express his knowledge, because it is obvious that it can’t do so by using \( A \alpha \) or \( \alpha \) (then the knowing subject would only refer to its own world and not to the real one). To give the problem a more general formulation: How can a knowledge that refers to another possible world \( w_2 \) be expressed in a possible world \( w_1 \)?

The only possibility Williamson seems to consider is that the knowing subject in \( w_1 \) uses an expression of the kind “in \( w_2 \), \( \alpha \)”. Thus, the knowing subject first has to specify or name world \( w_2 \), and then assert about this so specified world, that in it \( \alpha \) holds. Williamson discusses four candidates that might perhaps allow for a specification of a different possible world:

(i) by necessary and sufficient conditions  
(ii) by counterfactuals  
(iii) by space-time-coordinates  
(iv) by ostension

Clearly, (iii) and (iv) won’t do the job, because with the help of them one is not able to exceed the realm of one’s own world. Williamson’s arguments concerning the candidates (i) and (ii) run similarly and are not much different. Therefore, we will give in the following a reconstruction of his rather technical and difficult argument
against (i), which surprisingly amounts to the thesis that knowledge of the kind “in $w, \alpha$” consists of mere trivial logical knowledge as soon as $w$ has been specified by a necessary and sufficient condition (we will call this condition $\beta$ in the following).

After having convincingly explained that “in $w, \alpha$” amounts to the same as $\Box (w \text{ obtains } \rightarrow \alpha)$, Williamson presents his argument in a rather compact form:\textsuperscript{34}

Assume first that, in knowing that in $s, p$, one can specify $s$ in way (i). Thus, for some value of ‘$q$’, necessarily, $s$ obtains if and only if $q$: moreover, the knowledge that, necessarily, if $q$ then $p$ counts as knowledge that, in $s, p$. Now it is easy to show that, necessarily, $s$ obtains if and only if both $p$ and $q$. Thus the condition that both $p$ and $q$ specifies $s$ in way (i) just as well as the condition that $q$ does. It would therefore be quite ad hoc not to permit its use in de re knowledge about $s$, since the latter condition may be so used. Hence the knowledge that, necessarily, if both $p$ and $q$ then $r$ counts as knowledge that, in $s, r$. In particular, the knowledge that, necessarily, if both $p$ and $q$ then $p$ counts as knowledge that, in $s, p$. Thus, given the assumption about (i), the knowledge that, in $s, p$ requires no more than knowledge of a trivial logical truth. (Williamson 1987, 259)

The argument is difficult and a detailed reconstruction will be helpful. Let’s assume that $\alpha$ is really true in $w$, thus

\[ \Box (w \text{ obtains } \rightarrow \alpha), \]  

(14)

and ask what a knowledge of this fact must consist of. According to the reflections above (specially variant (i)), the knowing subject first has to specify $w$ by a necessary and sufficient condition $\beta$, and further has to know that this condition necessarily implies $\alpha$:

\[ \Box (w \text{ obtains } \leftrightarrow \beta) \]  

(15)

\[ \begin{array}{c}
\text{\textcolor{red}{\checkmark}} \Box (\beta \rightarrow \alpha).
\end{array} \]  

(16)

But, if $\alpha$ is true in $w$ and if $\beta$ is a necessary and sufficient condition for the obtaining of $w$, then:

\[ \Box (\beta \leftrightarrow (\beta \land \alpha)) \]  

(17)

And thus, also $\beta \land \alpha$ is a necessary and sufficient condition for the obtaining of $w$:

\[ \Box (w \text{ obtains } \leftrightarrow (\beta \land \alpha)) \]  

(18)

And consequently also $\beta \land \alpha$ can be used in both clauses for knowledge of “in $w, \alpha$”:

\[ \Box (w \text{ obtains } \leftrightarrow (\beta \land \alpha)) \]  

(15')

\[ \begin{array}{c}
\text{\textcolor{red}{\checkmark}} \Box ((\beta \land \alpha) \rightarrow \alpha).
\end{array} \]  

(16')
The knowledge required by (16‘) is mere trivial logical knowledge, and thus know-
ing “in $w, \alpha$” reduces to being able to specify $w$ by a necessary and sufficient con-
dition. But then, the knowledge that in $w, \alpha$ can no longer be distinguished
from the knowledge that in $w, \gamma$ (for any $\gamma$ different from $\alpha$ and holding in $w$).
As soon as one is able to specify a possible world (by a necessary and sufficient
condition) one knows all the truths holding in this world.

How to make this surprising consequence comprehensible? In order to specify
a world by a necessary and sufficient condition, this condition has to guarantee
that the world can be distinguished from all other possible worlds at hand of this
condition. But this can only be done by a condition that comprises all truths of this
world (metaphorically such a necessary and sufficient condition for the obtaining
of $w$ can be thought of as something like a big, maybe endless, conjunction of all
truths of $w$). Because, if the condition $\beta$ would not determine for a certain (conti-
gent) proposition $\alpha$ whether it is true or false in the possible world to be specified,
there would be two metaphysically possible worlds meeting the condition $\beta$, and
the crucial proposition $\alpha$ being true in one world and false in the other.

Williamson’s argument shows that up to now the sequence of symbols $\Diamond K A \alpha$
(or $\Diamond K^* \alpha$ in $S_5^*$, respectively) has not yet been given a sensible interpretation.
Because we still lack a convincing answer to the question how the non-actual
knowing subject might even express its knowledge: By $\alpha$ and $A \alpha$ it refers to its own
non-actual world, and the expression of its knowledge with a construction like “in
$w, \alpha$” (where $w$ designates the real world) fails, because such a knowledge reduces
to trivial logical knowledge, an unacceptable consequence. Williamson draws the
conclusion that as long as no justifiable conception of non-actual knowledge of
$A \alpha$
(which also has to determine how the non-actual knowing subject can express its
knowledge) has been given the scheme (1') (or (1*) respectively) “should be treated
as uninterpreted formalism” (Williamson 1987, 261).

4. A New Conception of Possible Knowledge

First, let’s come back to the standard understanding of (ART) via thesis (1). What
(1) requires is that if $\alpha$ is true in the real world there has to be a possible alternative
$w'$ in which $\alpha$ is also true and someone knows it (see Figure 2).

Concerning necessary a priori knowable truths this picture is perfectly accept-
able, because then the $\alpha$ in the one possible world and the $\alpha$ in the other possible
world don’t differ with respect to what is required from the respective inhabitants
of the corresponding worlds in order to know $\alpha$.35 Things look different when we
are concerned with contingent truths (or necessary, but only a posteriori knowable
truths), because then the knowing subjects have to know something (for example
via experience) that might be different between the two possible worlds.

An example: Take $\alpha$ to be “The President of the United States is male”. In our
real world in order to know $\alpha$ one has to have had experiences that are somehow
related to George Bush (to know $\alpha$ is knowing something of Bush), whereas in another possible world, in which Al Gore would have become president, one needs experiences that are somehow related to Al Gore in order to know $\alpha$ (to know $\alpha$ is to know something of Gore).

That’s the deeper reason why we rejected schema (1) in the beginning and replaced it with schema (1$^*$): It is not at all clear that someone in one possible world having knowledge which he expresses by $\alpha$ and someone in another possible world having knowledge which he expresses by $\alpha$, know ‘the same’. As the example above shows, their knowledge might be of different objects, a very good reason for assuming that they are not knowing ‘the same’.

On the other hand there was a problem with (1$^*$), too. As Williamson’s argument shows, the schema can’t be understood in a way, such that what is required is the knowledge that $\alpha$ is true in another possible world (see Figure 3).

What we are still in need of is a conception of how to understand non-actual knowledge of $\alpha$, where $\alpha$ refers to the actual world. The main idea of the proposal I want to make can be seen by means of the following picture: a non-actual knowing subject knows $\alpha$ ($\alpha$ referring to the real world), if it has knowledge (about its own world) that it can express by $\beta$ and $\alpha$ (with respect to $w$) and $\beta$ (with respect to $w'$)
express ‘the same’. To determine more exactly what it is that is ‘the same’ we need some more preparations (see Figure 4).

![Diagram](attachment:image.png)

\[ \text{Figure 4.} \]

4.1. **Knowledge de re and Knowledge de dicto**

Suppose that the American basketball fan Bill knows that in the game between the Dallas Mavericks and the Portland Trailblazers Dirk Nowitzki was the top-scorer. In fact, Dirk Nowitzki is the best German player in the NBA. But, Bill does not know that (let’s say, Bill thinks that Nowitzki is a Russian, because of the name, and thus still considers Detlef Schrempf to be the only German NBA-player, even if Schrempf has retired meanwhile and Shawn Bradley got a German passport: well, Bill is kind of an uninformed Basketball fan). What about Bill’s knowledge that the best German NBA-player scored the most in that game? In one sense of the word he has that knowledge because he knows that Nowitzki has been the highest scorer and Nowitzki is in fact identical with the best German NBA-player. But in another sense, Bill does not have the knowledge in question (ask him, whether the best German player has scored most, and his answer will be “No”), because he does not know that the proper name “Dirk Nowitzki” and the definite description “the best German NBA-player” denote the same person.

Knowledge in the first sense is called *de re* knowledge, knowledge in the second sense *de dicto* knowledge.\(^{36}\) If we apply this terminology to our example, it follows that Bill has the *de dicto* knowledge that Dirk Nowitzki was the top-scorer but he has not the *de dicto* knowledge that the best German NBA-player was the top-scorer. But, Bill has *de re* knowledge of the best NBA-player (who is Dirk Nowitzki) to have been the top-scorer.

In order to determine more exactly the notions of *de dicto* and *de re* knowledge, let’s consider first only knowledge concerning elementary propositions. Take an elementary proposition of the form \( F a_1, a_2, \ldots, a_n \) containing a predicate \( F \) and \( n \) singular terms \( a_1, a_2, \ldots, a_n \). We will apply some terminological proposals from Perry. He defines the *subject matter* of such a proposition as follows:

The *subject matter* of a sentence. This is the objects (or conditions) designated by the terms in the sentence, and the condition designated by the condition word in the sentence. (Perry 2001b, 147)\(^{37}\)
Thus, the subject matter of $Fa_1, a_2, \ldots, a_n$ is the set containing the relation designated by $F$ and the objects named by $a_1, a_2, \ldots, a_n$. Now, we can develop the concept of subject matter content. The subject matter content of a proposition says what has to be the case with respect to the subject matter so that the proposition is true. Namely in our case, that the objects named by $a_1, a_2, \ldots, a_n$ stand to each other in the relation designated by $F$. De re knowledge is nothing else as knowledge of the subject matter content, and this means knowledge of the existence of certain states of affairs or facts.\textsuperscript{38}

The subject matter content is distinguished from the reflexive content of a proposition by Perry.\textsuperscript{39} While it is irrelevant with respect to the subject matter content how for example the objects of the subject matter are named (the only thing that matters is which objects are named), this relation between language and world is decisive for the reflexive content. Because it determines the truth-conditions concerning the relation between language and world that have to be fulfilled so that the proposition is true. The reflexive content of for example $Fa_1, a_2, \ldots, a_n$ reads as follows: “The expression ‘$a_1$’ names an object $d_1$, the expression ‘$a_2$’ names an object $d_2$, \ldots, the expression ‘$a_n$’ names an object $d_n$, and ‘$F$’ designates a relation $R$ in such a way that the objects $d_1, d_2, \ldots, d_n$ stand in the relation $R$.”

De re knowledge can be conceived of as knowledge of the subject matter content of a proposition and de dicto knowledge can be conceived of as knowledge of the reflexive content of a proposition. But what about the relation between de re and de dicto knowledge? A person has de re knowledge that $\alpha$ if and only if there is a $\beta$ having the same subject matter content as $\alpha$ and the person has the de dicto knowledge that $\beta$. A person has no de re knowledge that $\alpha$ if and only if there is no such $\beta$.

A few words remain to be said regarding de re and de dicto knowledge concerning complex propositions. In order to avoid problematic entities like complex states of affairs we simply define de re knowledge as above via de dicto knowledge, and the subject matter content of complex propositions is determined in the usual way via the subject matter contents of their contained elementary propositions. For example, the subject matter content of a conjunction $\alpha \land \beta$ says that concerning the subject matter of $\alpha$ it has to be as $\alpha$ says and concerning the subject matter of $\beta$ it is to be as $\beta$ says for the whole conjunction to be true.

4.2. Quantifiers

It is common usage in logic to work with unrestricted quantifiers, that range over the whole universe of discourse, only. This is so because a formula containing a restricted quantifier ranging over a certain specific domain $D$ is always logically equivalent to a formula containing an unrestricted quantifier instead, for example:

$$\forall x \in D \alpha \iff \forall \alpha (x \in D \rightarrow \alpha)$$
But for our purpose of defining *de re* knowledge in terms of *de dicto* knowledge of propositions containing quantifiers it is necessary to use restricted quantifiers again. The subject matter of a general proposition of the form $\forall x \in D \alpha$ is constituted by the subject matter of $\alpha$ plus the set that is the domain of the quantifier, here $D$. And the subject matter content of $\forall x \in D \alpha$ says that for every object $d \in D$ named by $\tau$ it has to be true that $\alpha\left[\tau/x\right]$.40

To sum up our conception of *de re* knowledge we can say that it is build by the construction of equivalence classes of *de dicto* knowledges, and a certain equivalence class is constituted of those *de dicto* knowledges that are indifferent with respect to the subject matter content.

Examples: If Bill knows that Dirk Nowitzki was the top-scorer and if I know that the best German basketball player was the top-scorer, two different *de dicto* knowledges are involved, but nevertheless we have the same *de re* knowledge. With ordinary language examples concerning knowledge of quantified propositions we have to be careful:

*All philosophy students passed their logic exam.*  
(E)

Suppose that Bill knows (E) in the *de dicto* sense. Then it depends on our logical analysis of (E) which *de re* knowledge we ascribe to Bill. If we analyse (E) as $\forall SxPx$ (with $Sx$ standing for “x is a philosophy student” and $Px$ abbreviating “x passed his exam”) we attribute to Bill that he knows *de re* of all philosophy students that they passed their exam, but if we analyse (E) as $\forall x(Sx \rightarrow Px)$ we ascribe to Bill the *de re* knowledge of everything that if it is a philosophy student it passed its exam. The first alternative (as in almost all similar cases) seems to be the more natural one.

4.3. SAME *DE RE* KNOWLEDGE IN DIFFERENT POSSIBLE WORLDS

The question arises what kind of knowledge is at stake in Fitch’s Paradox. As already argued above, to claim that two persons in two different possible worlds know ‘the same’ if they have the same *de dicto* knowledge is a little bit problematic because it might be that their knowledge is even of different objects (as in the Bush/Gore-example). I propose that the knowledge operator in Fitch’s Paradox should be understood as expressing *de re* knowledge (with respect to *a posteriori* truths):

$$\alpha \rightarrow \diamond k_{de re}^* \alpha$$  
(1*)

Finally, a very simple example will illustrate the conception and at the same time give a hint how Fitch’s Paradox is now resolved: Suppose that there are only two objects in the real world $w$, namely the two persons Tom and Bob, and that both are stupid (to be stupid means here to know nothing at all). Thus, in $w$ the following proposition is true:

*All are stupid.*  
($\alpha$)
Certainly, $\alpha$ isn’t known in $w$, because Tom and Bob are both stupid, and thus know nothing at all. And there is also no possible alternative in which $\alpha$ is knowable de dicto. Nevertheless there is a possible world in which somebody knows de re what $\alpha$ expresses with respect to the real world, namely that all inhabitants of $w$ (Tom and Bob) are stupid. Take for example the world $w'$ with its only three objects Tom, Bob and Jim. Tom and Bob, again, are stupid, but Jim is not, because he knows (at least) that Tom and Bob are stupid. For example he might express his knowledge by “All but me are stupid” or “Tom and Bob are stupid”. Thus, Jim in world $w'$ has the de re knowledge that $\alpha$ ($\alpha$ referring to the real world), even if he himself is not able to express his knowledge by $\alpha$ (see Figure 5).

5. Conclusion

Aim of this paper has not been to answer the question whether all truths might possibly be known. Whether the thesis is correct cannot be decided given the arguments presented here, alone. But I hope to have shown that Fitch’s argument is not able to decide the question against (ART), either. The position of soft anti-realism stays untouched by Fitch’s Paradox if the anti-realist is willed to give (ART) the reading (1*) within $S5^*$ (or (1’) within modal logic with actuality-operator, respectively) and to understand the kind of knowledge involved in the thesis as de re knowledge in the sense developed here in order to deal with non-actual knowledge of $\alpha$ ($\alpha$ referring to the actual world).41

Notes

1 This citation only talks about the meaning of a mathematical proposition, but a little later Dummett withdraws from this restriction:

The argument involved only certain considerations within the theory of meaning of a high degree of generality, and could, therefore, just as well have been applied to any statements whatever, in whatever area of language. (Dummett 1978, 226)
The epistemic constraint that every truth must be knowable under adequate (counterfactual) circumstances may be too strong and should be replaced by a weaker one claiming that for every truth there must possibly be good reasons to believe it or that there must possibly be accessible evidences for every truth. But with respect to the following arguments the differences between stronger and weaker formulations of the anti-realistic thesis in this sense should be of no relevance.

Here, as in the following, for the sake of simplicity the anti-realistic thesis is not formulated as a necessity claim, which could easily be done by adding a necessity operator: \( \Box (\alpha \rightarrow \Diamond K \alpha) \). Even if such a modal rendering is adequate (for the anti-realist the epistemic constraint of the concept of truth is a necessary one), the chosen simplification should have no effect on the following arguments.

The argument really originates with a referee of an unpublished paper by Fitch in 1945. Thus the name “Fitch’s Paradox” does not seem to be fully justified, but the real originator is unknown (at least to me).

It should be conceded that there exist certainly lots of full-blooded realists who consider already the anti-realistic thesis to be (almost) absurd, and who take Fitch’s Paradox only as a further confirmation of their anyhow existing opinion.

A short note on other approaches: (Wansing 2002) proposes to use a modal epistemic logic based on Nelson’s constructive propositional logic with strong, constructive negation. In such a logic modus tollens fails and thus the step from (5) to (6) in the derivation of Fitch’s Paradox is no longer justified.

(Dummett 2001) argues that the anti-realistic thesis should be restricted to basic statements, because the property of knowability is not transmitted from atomic to complex formulas: Even if \( \alpha \) is knowable and if \( \beta \) is knowable \( \alpha \land \beta \) doesn’t need to be so.

Proposals with some relation to the one presented later on in this paper (although not the same) can be found in (Kvanvig 1995), (Lindström 1997) and (Rabinowicz and Segerberg 1994).

A number of issues coming up in my paper are discussed in Williamson (2000a, 270–301).

Sure, there might be several non-human subjects which, so to say, could distribute the knowledge of all truths among each other. But then, according to a result by (Humberstone 1985) the class of knowing subjects needs to be infinite, because if for a finite group of individuals every truth is known by someone in the group, then there is someone in the group who knows every truth. And thus, in the case of a finite group, again there would be at least one omniscient subject.

Probably some theists might insist that God actually is a (possible) member of our communication community (think of prayers and the like). But I can’t and I don’t want to engage in such a debate here.

Thus, (Usberti 1995) proposes to restrict thesis (1) as referring only to mathematical propositions. Then it is no more possible that the knowledge operator \( K \) appears in \( \alpha \), and thus constructions as the one used in the derivation of Fitch’s Paradox are automatically excluded. But, a restriction of its theses to mathematics is no longer in the spirit of modern anti-realism, a position which tries to generalise ideas that had been developed in the field of the philosophy of mathematics to other areas. The drastic restriction proposed by Usberti concerning the range of \( \alpha \) in thesis (1) amounts, in the end, to give up the position of anti-realism.

Williamson, the most eager defender of Fitch’s Paradox against anti-realistic proposals, seems to think that the possibilities arising from a replacement of classical logic by intuitionistic logic may be the only rescue for the anti-realistic position:

How [...] might a verificationist escape from Fitch’s argument? One way would be to substitute intuitionistic for classical logic. It may even be the only way. (Williamson 1987, 261)

The more general conception of the intuitionistic conditional saying that the assertion of a conditional \( \alpha \rightarrow \beta \) is justified whenever the justified assertability of \( \alpha \) entails the justified assertability of \( \beta \), leads to similar consequences.

A detailed discussion of the relevant consequences arising from a further conception of the intuitionistic conditional – namely the one which says that in an intuitionistic conditional \( \alpha \rightarrow \beta \) a prove
of \( \alpha \) has to be transformable into a proof of \( \beta \) (cf. for example Dummett (1977, 12–13) – can be found in Williamson (1988, 429–431).

12 For example, it could be argued that the domain of the general quantifier of the semantic condition for the intuitionistic conditional does not necessarily have to coincide with the domain of the existential quantifier implicit in the \( K \)-operator (ranging over the members of the linguistic community). Because then, knowledge of \( \alpha \) would not automatically entail knowledge of \( K \alpha \), because under these circumstances \( K \alpha \) even could be false.

13 Concerning the question how the position of moderately hard anti-realism can be further developed, see the proposals made in Williamson (1982, 1988, 1992).

14 It should be mentioned that within the framework of intuitionistic logic there have been made other proposals dealing with restrictions of thesis (1). For example, (Tennant 1997), using his intuitionistic relevance logic \( IR \), proposes to restrict the range of thesis (1) to ‘cartesian’ propositions. And a proposition \( \alpha \) is cartesian if and only if no contradiction follows from \( K \alpha \). (Tennant 2001) answers the critics of (Hand and Kvanvig 1999) that the proposed restriction being \( ad \ hoc \), but (Williamson 2000b) sharp-wittedly shows that Tennant’s proposal does not succeed in avoiding the problematic consequences it wants to get rid of.

15 For the sake of simplicity we suppose that the sentences in question are not necessary truths (or falsities, respectively). Of course, concerning necessary truths (or falsities, respectively) it can be supposed that they are true (or false, respectively) at all points of time.

16 Explanation: It is not possible that \( t_1 > t_0 \) because \( t_0 \) is the present point in time and thus in case of \( t_1 > t_0 \) the antecedent would lack a truth-value. On the other hand, \( t_0 > t_1 \) isn’t possible either, because the formula inside the parentheses refers to \( t_0 \), and thus it does not have a truth-value before \( t_0 \).

17 Because if \( t_2 \) would lie in the future \( K_{t_2}(\alpha_{t_0} \land \neg K_{t_0} \alpha_{t_0})_{t_1} \) would not have a truth-value yet.

18 That means, the accessibility relation \( R \) is such that \( w_0 R w_1 \).

19 Such \( S4 \)-structures as the one defined by (C1), (C2) and (C3) seem to be very natural when dealing with temporal issues.

20 For a detailed, but in parts rather difficult, analysis of this thesis as well as of further questions concerning the relation between truth-values and time-parameters that arise from an anti-realistic point of view, see the chapter Anti-Realism, Timeless Truth and Nineteen Eighty-Four in Wright (1993, 176–203). A classical source for the discussion of these and similar problems is the paper The Reality of the Past in Dummett (1978, 358–374).

21 The discussion of Melia’s analyses and arguments in the main text is maybe too goodwilling, because the formulations in the two citations are very problematic. On the one hand, I think, Melia makes a mistake when he demands that one only has to \( try \) to find out the truth-value of certain propositions to cause a ‘change’ of it. This demand certainly is too weak because in the problematic cases (as for example the one with the Fitch formula) the truth-value necessarily ‘changes’ only if these efforts are successful. But, what should it mean to ‘find out’ something that is ‘changed’ by this ‘finding out’ itself? It seems to me that the talk of the ‘change’ of truth-values in this context doesn’t amount to more than the fact that the same (contingent) proposition may have two different truth-values with respect to two different possible worlds (but that’s trivial). And as there are propositions, as is shown by Fitch’s Paradox, that cannot be known when they are true, their truth-value can only be known in worlds in which they are false.

22 A proof that it is impossible to show the falsity of (13) by pure logical means can be provided using modal and epistemic possible worlds models in which some world \( w \) is modally accessible from every world but only \( w \) is epistemically accessible from \( w \).

23 Naturally, it is not excluded that there might be philosophical (non-logical) arguments that might favour a replacement of (ART) by (LART), or even philosophical (non-logical) arguments that might discredit the philosophical position of anti-realism altogether.
24 In ordinary language, the semantic role that is typically played by the subjunctive mood is sometimes articulated by other linguistic means. For a discussion of this point see Wehmeier (2003a).
25 For a detailed analysis of the example see Wehmeier (2003a, 7-8). Sure, that there are propositions that cannot be expressed within standard modal logic is well-known for quite a long time (for example, see Hazen (1976) and Crossley and Humberstone (1977)).
26 The standard way to deal with the expressibility problem is to introduce a so-called actuality-operator. For a discussion of the relation between Wehmeier’s proposal and modal logic with actuality-operator, see below.
27 Should the formal language be enriched by supplementary operators, there has to be an indicative as well as a subjunctive version also of those, as for example of the knowledge operator (K and K∗). It is possible to work with several subjunctive markers in order to distinguish different binding relations in encapsulated modal contexts (but such subtleties are of no relevance for the aim of this paper). Furthermore, Wehmeier limits his discussion to S5∗, in which all possible worlds are accessible to each other. For corresponding versions of other modal logic systems that pose weaker conditions on the accessibility relation R (as for example T∗ or S4∗), there have to be indicative and subjunctive versions of the modal operators themselves.
28 One might wonder why the solution proposed to block the derivation of Fitch’s Paradox has been presented first in the new framework of S5∗ and not in the framework of the much better known modal logic with actuality-operator. But, I consider Wehmeier’s modal logic with subjunctive marker to be the better and more natural formalism compared to modal logic with actuality-operator for the reasons given later on and in Wehmeier (2003a).
29 As with Fitch’s Paradox, also here the identity of the first one to have made this proposal for a solution is unclear. Because, in a footnote Edgington reports that Lloyd Humberstone had independently reached essentially the same solution, and Williamson (1987) writes in the first footnote that he had been confronted with such a proposal already in 1982 by an anonymous referee.

At the same time Humberstone has published a paper (cf. Humberstone (1982)), in which he suggests a modal logic, which respects the semantically relevant differences between the indicative and the subjunctive mood by the help of a subjunctive operator. A discussion of the similarities and differences between this modal logic and his own can be found in Wehmeier (2003a, 12).

Finally, it should be mentioned that I discovered the presented blocking of the derivation of Fitch’s Paradox in S5∗ before having been acquainted with Edgington’s paper and the subsequent literature.
30 See for example Sorensen (1988) and Wright (1993).
31 The argument and the analogy between necessity propositions and general propositions is inspired by Wehmeier (2003a, 24–25). But, I think, his further thesis that propositions of the form □(a = b), or □(a =∗ b) in S5∗ respectively, are no real necessity statements, and thus identity statements are no potential candidates for necessary a posteriori truths, is misleading because in the scope of the modal operator there is a predicate that might be in subjunctive mood. That the difference between the indicative and the subjunctive mood does not really matter here depends on the peculiarities of the identity relation, I think, and not on the logical form of the proposition.
32 Here, I suppose that knowledge is principally mediated by language and that it is part of the truth-conditions of a proposition like “S knows α” that for the subject S it is at least in principle possible to express this knowledge from his perspective within his language.
33 Those acquainted with so-called hybrid logic (cf. for example Blackburn (2001) and Blackburn et al. (2001)) might think that using this kind of logic solves the problem of the specification of other possible worlds straightforwardly, because hybrid logic is essentially modal logic enriched with so-called nominals, formulas that are true in exactly one possible world and thus name possible worlds in a certain sense. The problem with this idea is that the hybrid logic used by the logician or philosopher in order to model counterfactual situations and modal discourse from an outside perspective is not necessarily available to the non-actual knowing subject, too. The knowing subject should be able to express its knowledge with the means available to it, mainly ordinary language, and I doubt whether
there is something in ordinary language corresponding to the nominals of hybrid logic (at least when hybrid logic is used to deal with metaphysical modality).

34 The s in the citation corresponds to our \( u \), the \( p \) to our \( \alpha \), and the \( q \) to our \( \beta \).

35 With respect to necessary a priori truths the anti-realistic thesis seems to be much less problematic, and maybe even uncontroversial. Thus, we will not consider such truths anymore in the rest of the paper.

36 The distinction between de re and de dicto knowledge and thus between de re and de dicto beliefs is heavily discussed in the literature. Often it is thought of to be a genuine distinction: a certain belief is either de re or de dicto (see for example Burge (1977)). I will not use the distinction that way. According to my conception of the distinction it is a matter of different perspectives: From the perspective of the knowing person each knowledge has to be formulated in a certain way, and thus each knowledge is de dicto in the first place. De re knowledge is a construction from an outside perspective, if one abstracts from how the objects, properties and relations the knowledge is about are designated in the de dicto knowledge. Thus, no de re knowledge without de dicto knowledge. True, there are problems if indexicals as for example “here” are involved because then what is known depends on the context. But, I think, this does not speak immediately against the conception that assumes that all knowledge is in the first place de dicto (for an attempt to deal with these difficulties see McDowell (1984)). Because these problems with indexicals are of no importance for the aim of this paper, I will assume for the following that no indexicals are involved. A classical source for the discussion of the difficulties that might evolve concerning the relation between de re and de dicto beliefs and indexicals is Perry (1979).

37 I have to thank Albert Newen (Bonn) for sending me Perry’s paper already before its publication.

38 Instead of applying Perry’s conception of subject matter content in order to specify de re knowledge it would have been possible to work with so-called Russellian propositions, too (cf. for example Barwise and Etchemendy (1987)).

39 The concept of reflexive content was introduced in Barwise and Perry (1983). (Perry 2001a) makes use of it to refute several counter-arguments against the thesis of the identity between the physical and the mental.

40 Similar considerations hold in the case of existential quantifiers.

41 I understand this paper as a tentative, initial presentation of a new idea how to solve Fitch’s Paradox of knowability. Certainly, some conceptions have to be examined in much more detail, especially the exact way of constructing equivalence classes out of items of de dicto knowledge in order to determine the de re knowledge in the case of the involvement of quantifiers. I have to thank Ulrich Nortmann (Saarbrücken), Shahid Rahman (Lille) and Kai Wehmeier (Irvine) for stimulating discussions. Mark Siebel (Leipzig), John Symons (El Paso), Heinrich Wansing (Dresden) and Timothy Williamson (Oxford) read an earlier version and provided many criticisms and remarks that lead to an improvement of the paper. The main ideas of this paper were presented at the philosophical colloquium of the University of Saarbrücken (December 2000) and at the Logic and Logical Philosophy 2001 workshop at Dresden (March 2001).

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